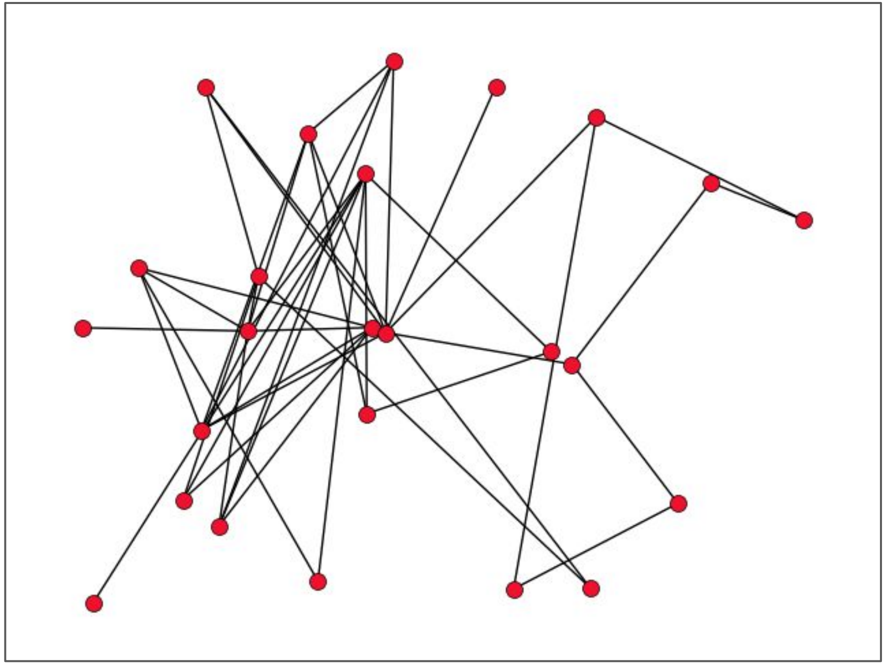
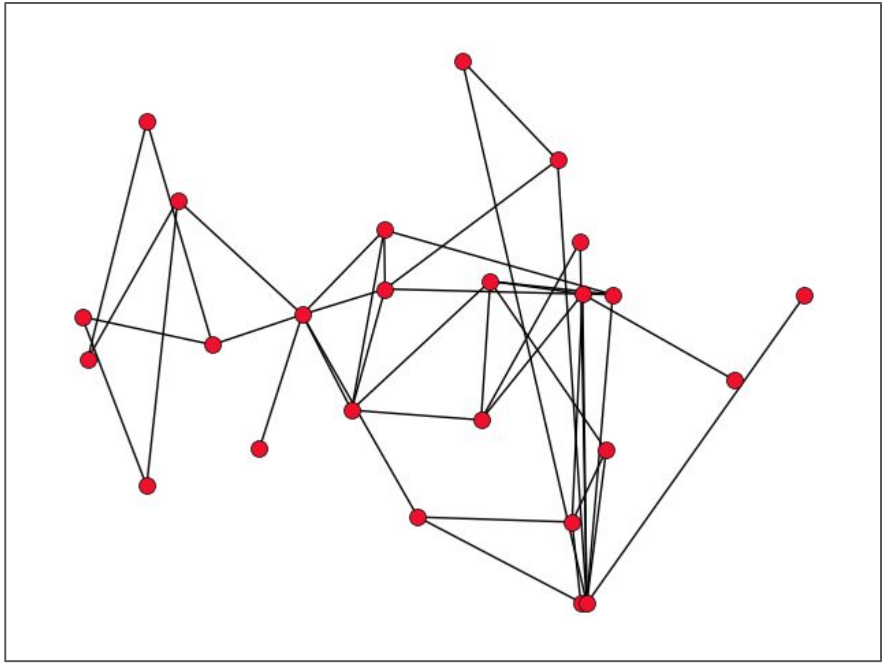
The Perception of Stress in Graph Drawings

Gavin J. Mooney[1], Helen C. Purchase[1], Michael Wybrow[1], Stephen G. Kobourov[2], Jacob Miller[2]

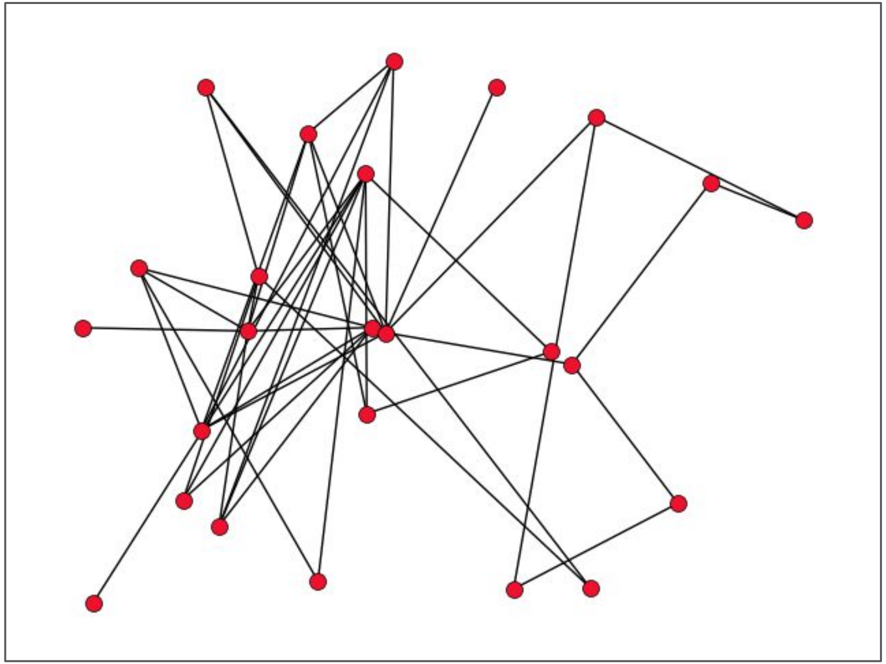
[1]Monash University; [2]Technical University of Munich

Which drawing has more crossings?

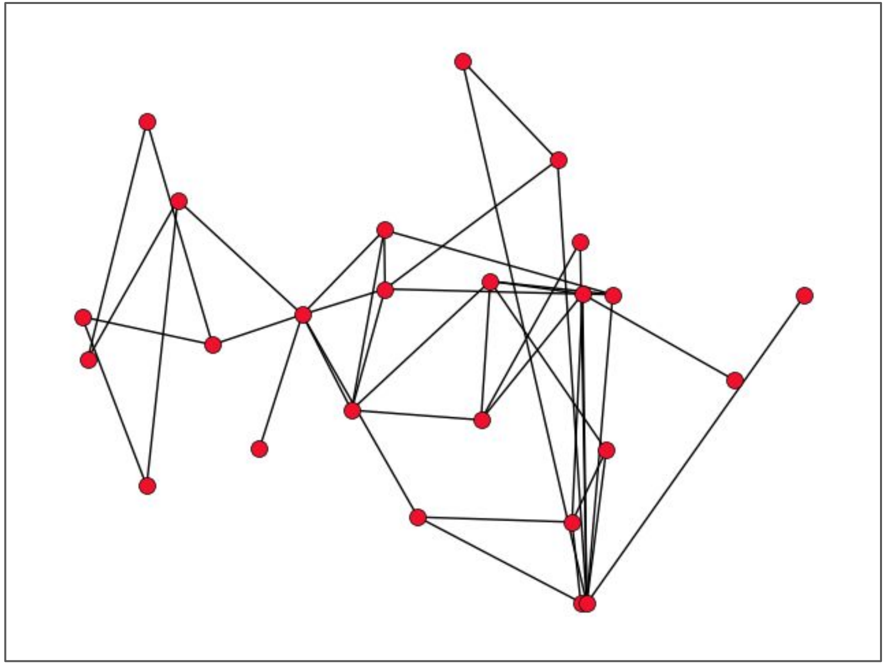




Which drawing has more crossings?

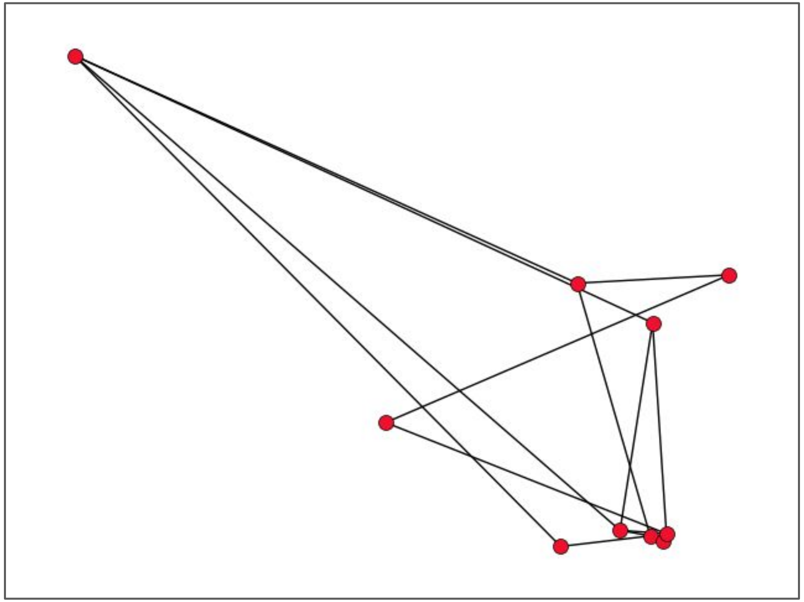


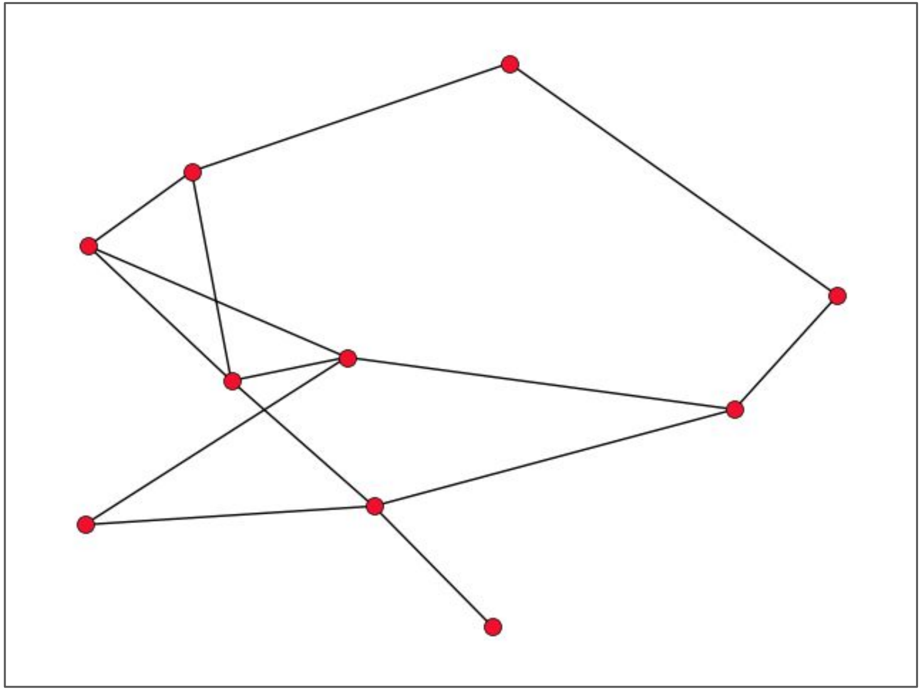
147 crossings



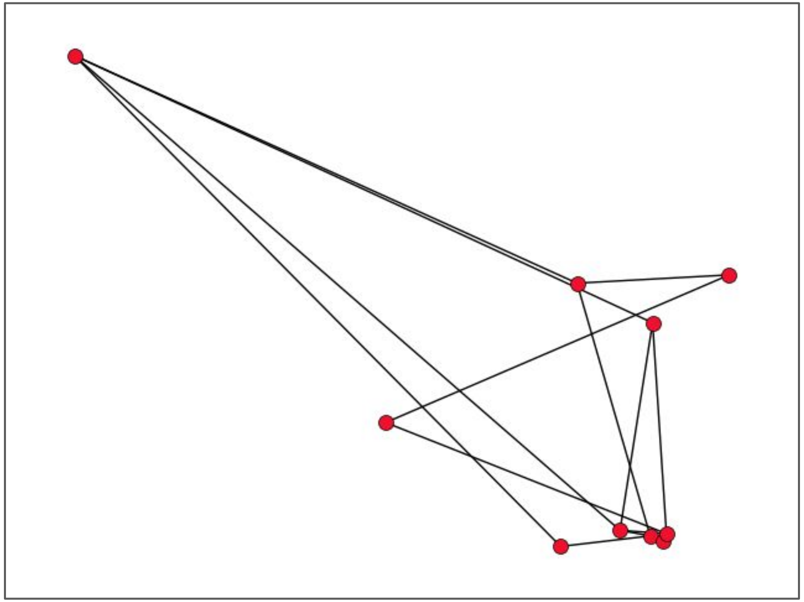
55 crossings

Which drawing has lower stress?

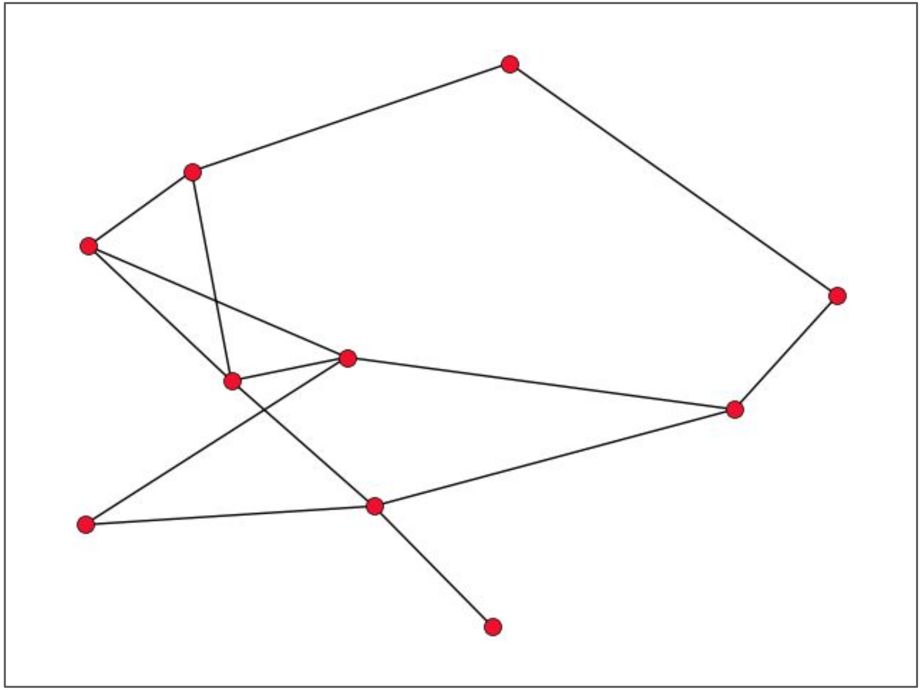




Which drawing has lower stress?

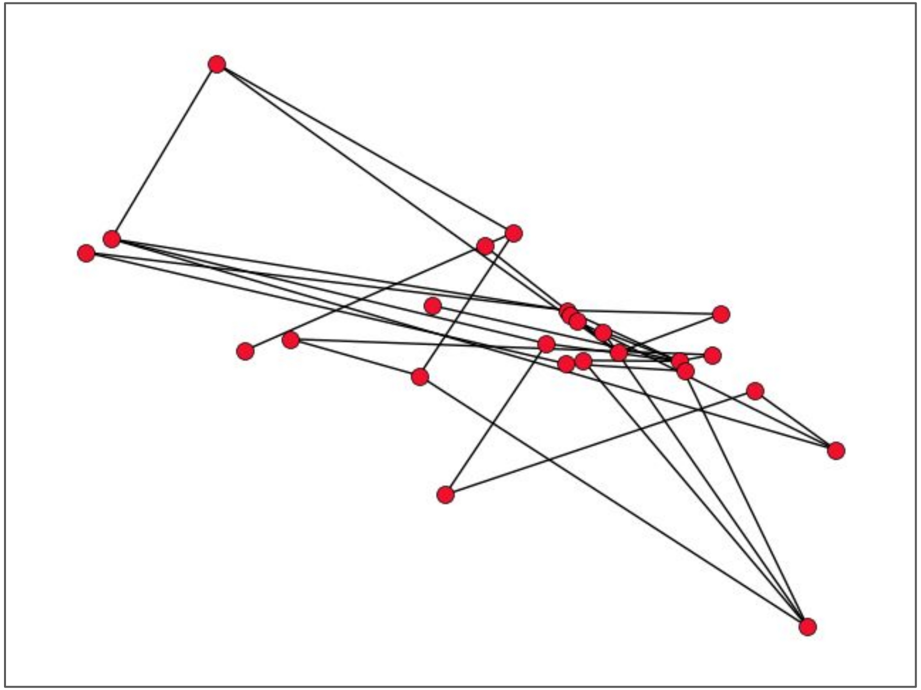


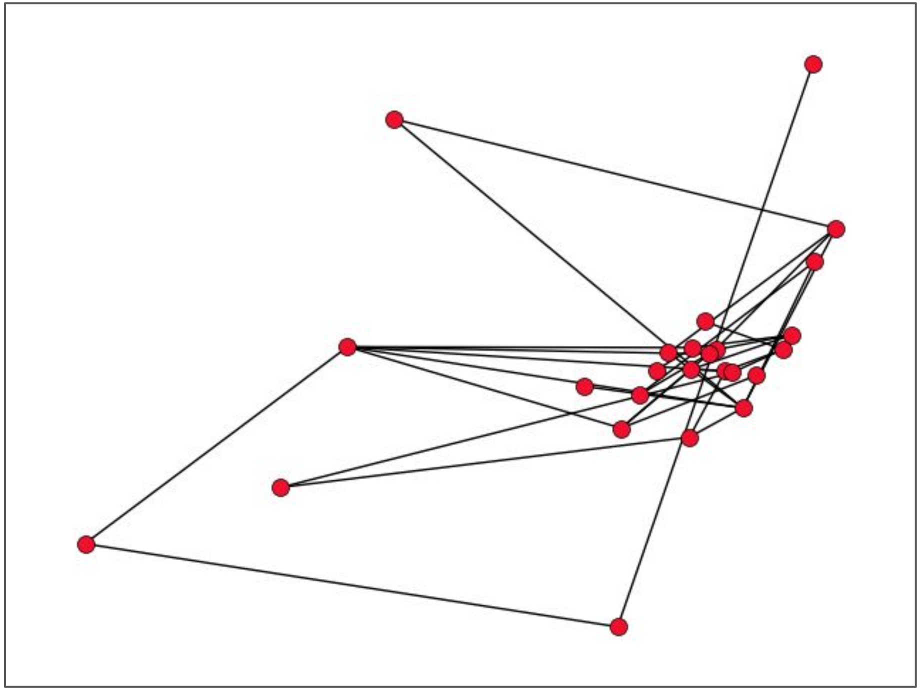
higher stress



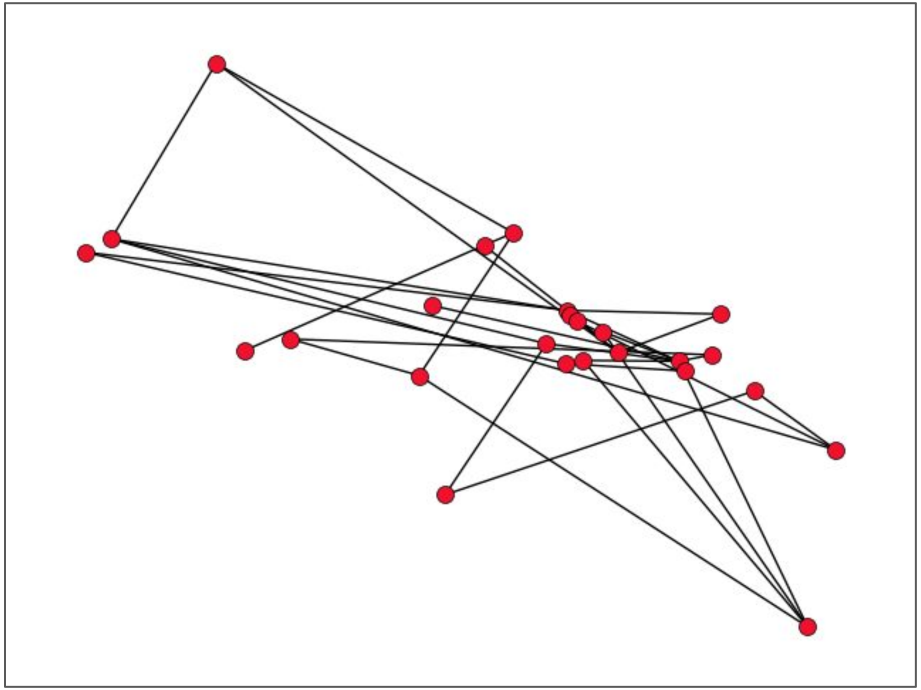
lower stress

Which drawing has lower stress?

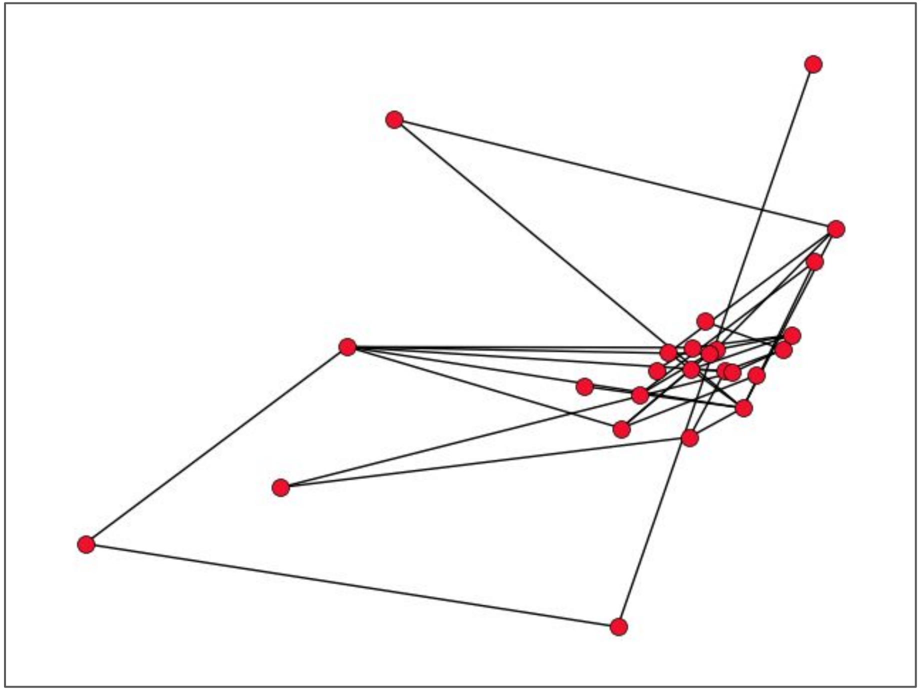




Which drawing has lower stress?



lower stress



higher stress

What is stress?

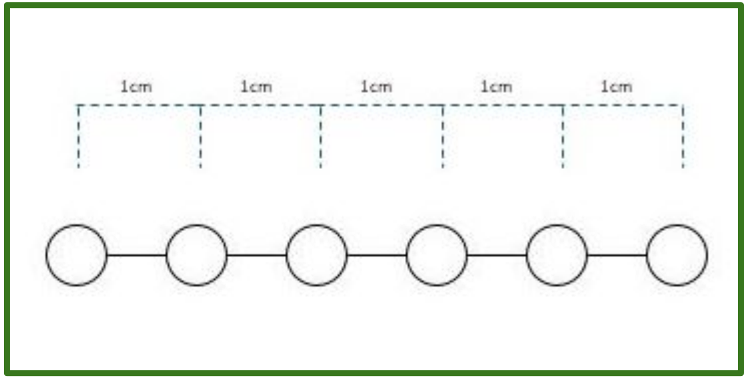
- The extent to which *graph theoretic* distances in a graph line up with *geometric* distances in the graph drawing.

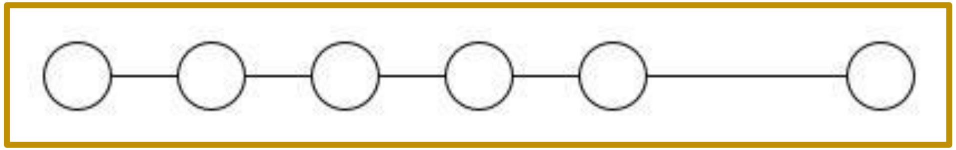
- Formally:

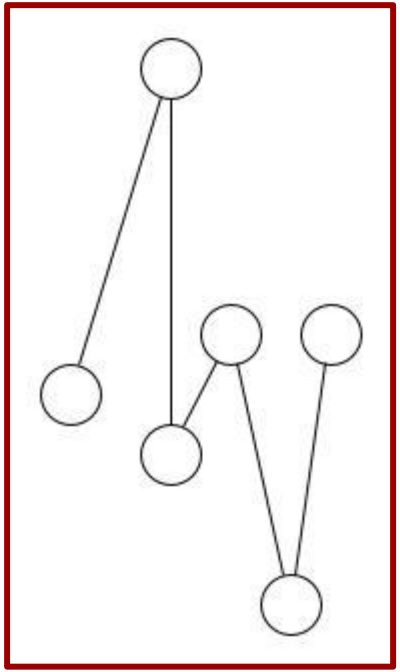
$$\sum_{i<j} \frac{(||X_i - X_j|| - d_{i,j})^2}{d_{i,j}^2}$$

Torgerson, W.S. Multidimensional scaling: I. Theory and method. *Psychometrika* **17**, 401–419 (1952). https://doi.org/10.1007/BF02288916

What is stress?







Kruskal Stress Metric (KSM)

Instead of calculating the exact discrepancy between theoretic and geometric distance:

Maintain the ordering of distances

i.e. from each node, the node that is first, second, and third etc. closest in terms of graph theoretic distance should still be first, second, and third etc. closest in the drawing respectively

$$\sqrt{\frac{\sum_{i<j}(||X_i - X_j|| - \hat{d}_{i,j})^2}{\sum_{i<j}||X_i - X_j||^2}}$$

Kruskal, J.B. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika* **29**, 1–27 (1964). https://doi.org/10.1007/BF02289565

Kruskal Stress Metric calculation

A Shepard diagram is formed by forming a set of coordinates for each pair of nodes i, j: $c_{i,j} = (||X_i - X_j||, d_{i,j})$.

A Shepard diagram created from an ideal drawing would have all points lying exactly on a straight line, since this would correspond to all input distances being exactly all output distances.

To measure the deviation from this line, Kruskal performs a monotonic regression with (n choose 2) points to best fit the diagram. The matrix D^ is defined such that $\hat{d}_{i,j}$ is the distance in the x-coordinate to the fitted line from $c_{i,j}$.

Kruskal Stress Metric (KSM)

Why are we using this definition?

- Scale invariant
- Rotation invariant
- Normalised
  - Values lie in the range 0-1, where 1 represents the “best” value, i.e. zero stress.
- Useful experimental alternative to traditional stress
  - Correlation of 0.871 on dataset of ~half a million graph drawings (KSM vs KK)

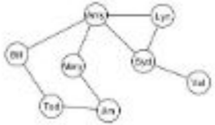
Experimental Design

- 3 Experimental conditions: *Trained, Untrained, Expert* participants
  - Recruited via prolific
- 3 Graph sizes: nodes: 10, 25, 50 - 5 graphs of each size
- *Trained*: 25 participants per graph size (75 total)
- *Untrained*: 10 participants per graph size (30 total)
- *Expert*: 8 participants - experts participate for all graph sizes

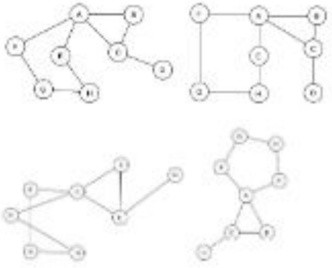
Training material - Explanation of stress

Definitions: networks and network drawings

A network is made up of objects and connections. For example, this social network depicts people (represented as circles) and friendships (represented as lines between the circles). Amy has four friends; Ted has two.



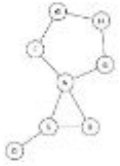
The same network can be drawn in many different ways by changing the position of the objects. For example, here are four drawings of the same network.



A 'path' is a series of steps between objects. For example, in the network below, the length of the path between G and F is 4; the length of the path between A and D is 2 or 3 (depending on whether you go through B or not).

The 'shortest path' is, as its name suggests, the shortest path when there is more than one way to get from one object to another.

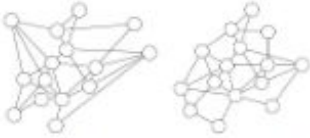
For example, in the network below, the shortest path between F and B is 2 (going through A, but not C or G/H/E); the shortest path between D and H is 4 (going through C/A, but not through B or F/G).



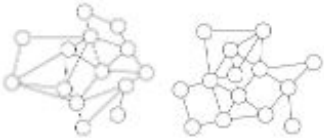
Definitions: visual properties of network drawings

Given that there is more than one way to draw a network, we can distinguish between them by their 'visual properties'.

For example, the drawing on the left has 'tighter angles' than the one on the right. These are both drawings of the same network.



And the drawing on the right has 'more symmetry' than the one on the left. These are both drawings of the same network.

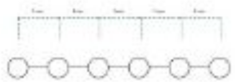


Training material - Explanation of stress

In this experiment, we are interested in the visual property of "stress".

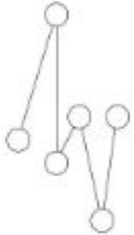
A drawing has low stress if the distance between pairs of objects is proportional to the length of the shortest path between them.

In its simplest form, the following network drawing has very low stress: the distance between each pair of objects is directly proportional to the (shortest) path between them.

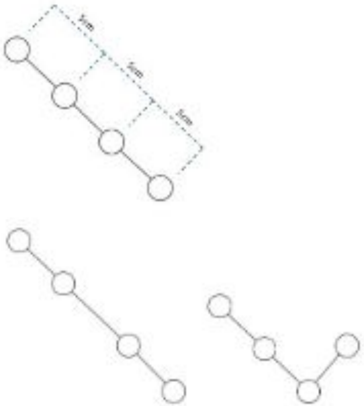


We just need to move one of the objects to increase the stress – the distance between the two objects at each end (6cm) is now longer proportional to the length of the path between them (5).

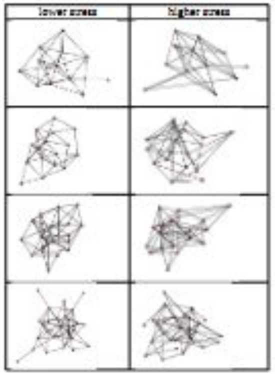
The same simple network can be drawn with even higher stress, where there is barely any relationship between the distance between the objects and the length of the paths between them.



Similarly, here is another network with very low stress, with two versions of higher stress.



Of course, this more-or-less-stress judgement becomes more difficult with larger networks. Here are some more examples showing the same networks drawn with different amounts of stress. In all cases, the network on the left has less stress than the network on the right. Thus, the network on the left maintains the distance/path relationship between pairs of objects better than the one on the right.



Of course, we cannot assess the differences in stress by doing all the distance and path length calculations in our head! But we can get a "feeling" as to when one drawing has less stress than another.

Before you start the experiment, we will ask you to make your own judgements, and let you know whether you are correct or not.

Training material

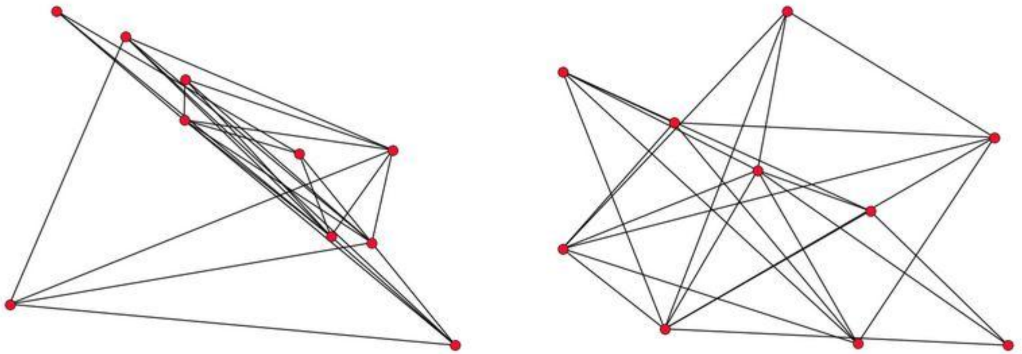
- Explanation of stress

- Participants were asked 9 training questions (similar to main experiment)
    - Shown two graph drawings and asked which has lower stress
        - The drawing on the **left** has lower stress
        - The drawings have the **same** stress
        - The drawing on the **right** has lower stress
    - Needed 5/9 correct to continue
    - *Trained* participants received accuracy feedback
    - *Untrained* participants given no accuracy feedback

- *Expert* participants were not given any training questions

Training material

* Do the two drawings have the same stress?

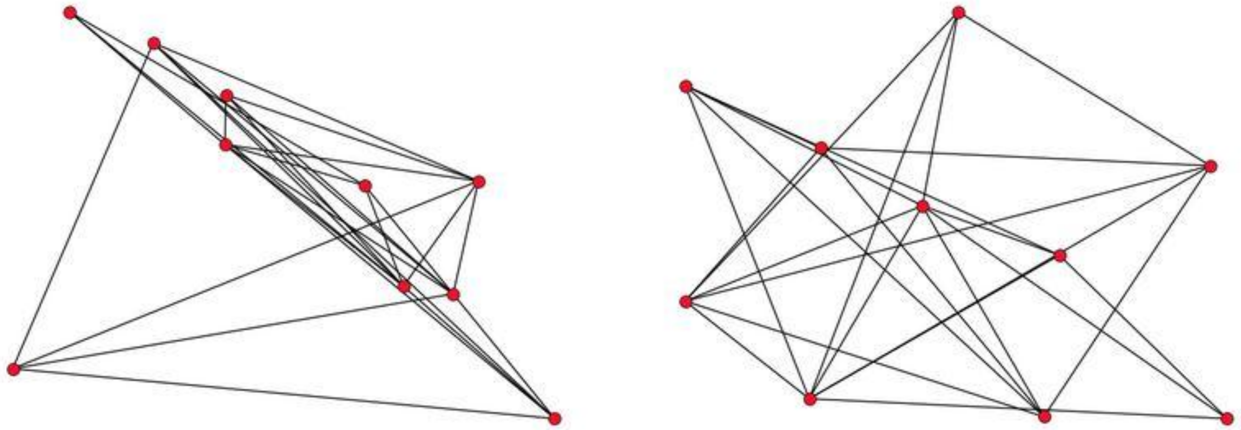


- [ ] The drawing on the **left** has lower stress
- [ ] The drawings have the **same** stress
- [x] The drawing on the **right** has lower stress

Next page >

Training material

Do the two drawings have the same stress?

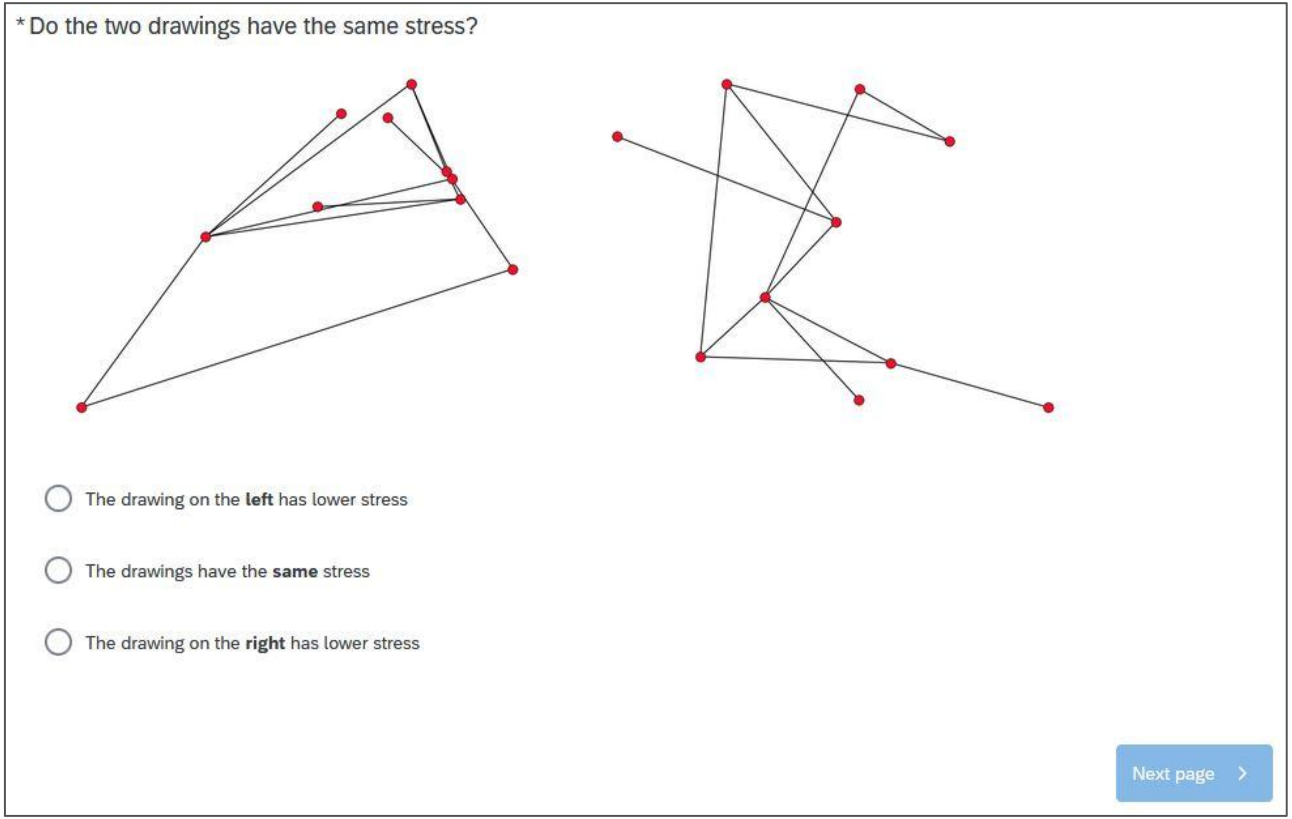


The previous response was ***correct***. The drawing on the **right** has lower stress.
Please continue with the survey.

Next page >

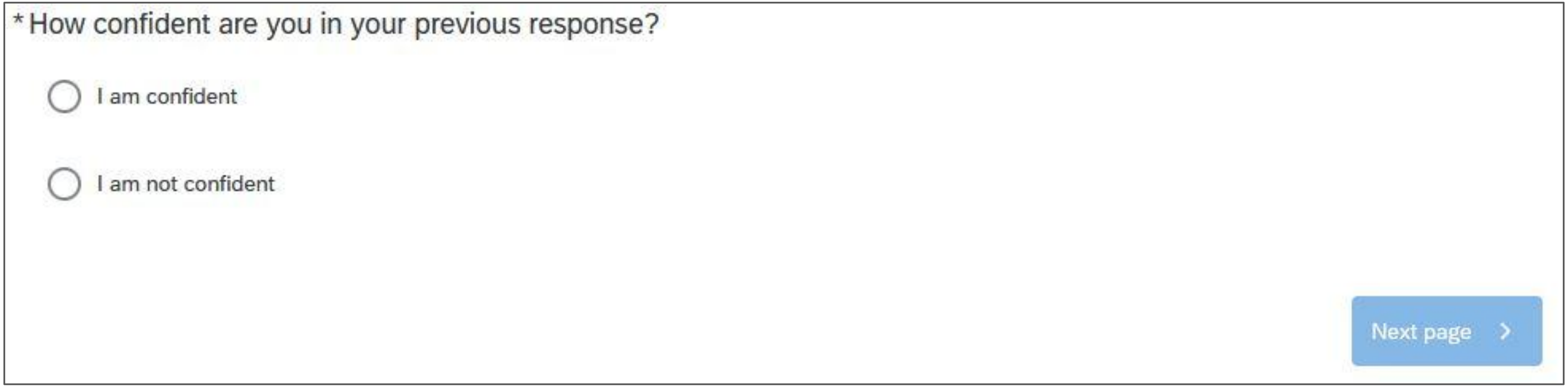
Experimental Procedure

- Compare two graph drawings, and pick the one with lower stress.
- Same graph; different drawings



Experimental Procedure - Data collected

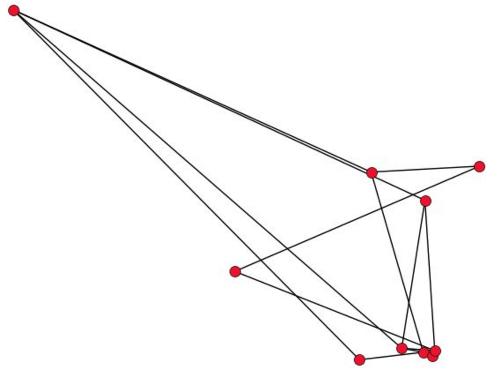
- **Accuracy - 0 or 1**
- **Response time - in seconds**
- **Confidence rating - 0 or 1**
- Strategy + demographic information



Stimuli - Why limit to 0.4 and 0.8 bounds?

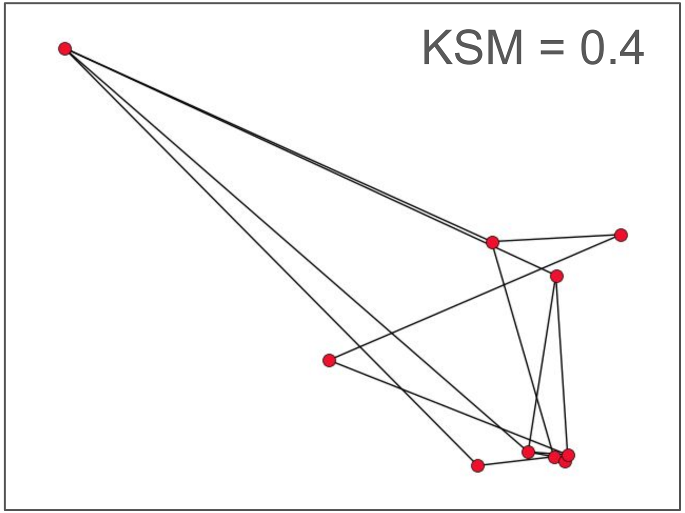
- Upper limit at 0.8 because the optimisation algorithm used is not capable of generating anything higher

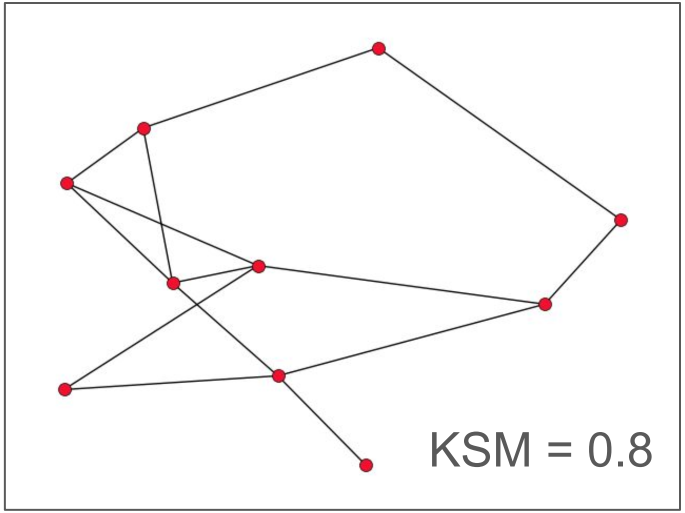
- Lower limit at 0.4 because anything lower creates drawings where one node is moved as far as possible from other nodes.
  - Similar to this drawing but more extreme



Stimuli

- 3 Graph sizes: 10, 25, 50
  - Low density graphs (n <= m <= 2n)
  - 5 Graphs for each size
  - Randomly generated using the Erdős–Rényi model
- Drawings generated with hill climbing algorithm for 9 distinct KSM values
  - 0.4, 0.45, 0.5... 0.8 (and hence 9 unique differences: 0, 0.05, 0.1, 0.15… 0.4)
  - 15 x 9 = 135 drawings
  - Repeated 3 times
- 3 sizes × 5 graphs × 9 stress values × 3 drawings = 405 total drawings





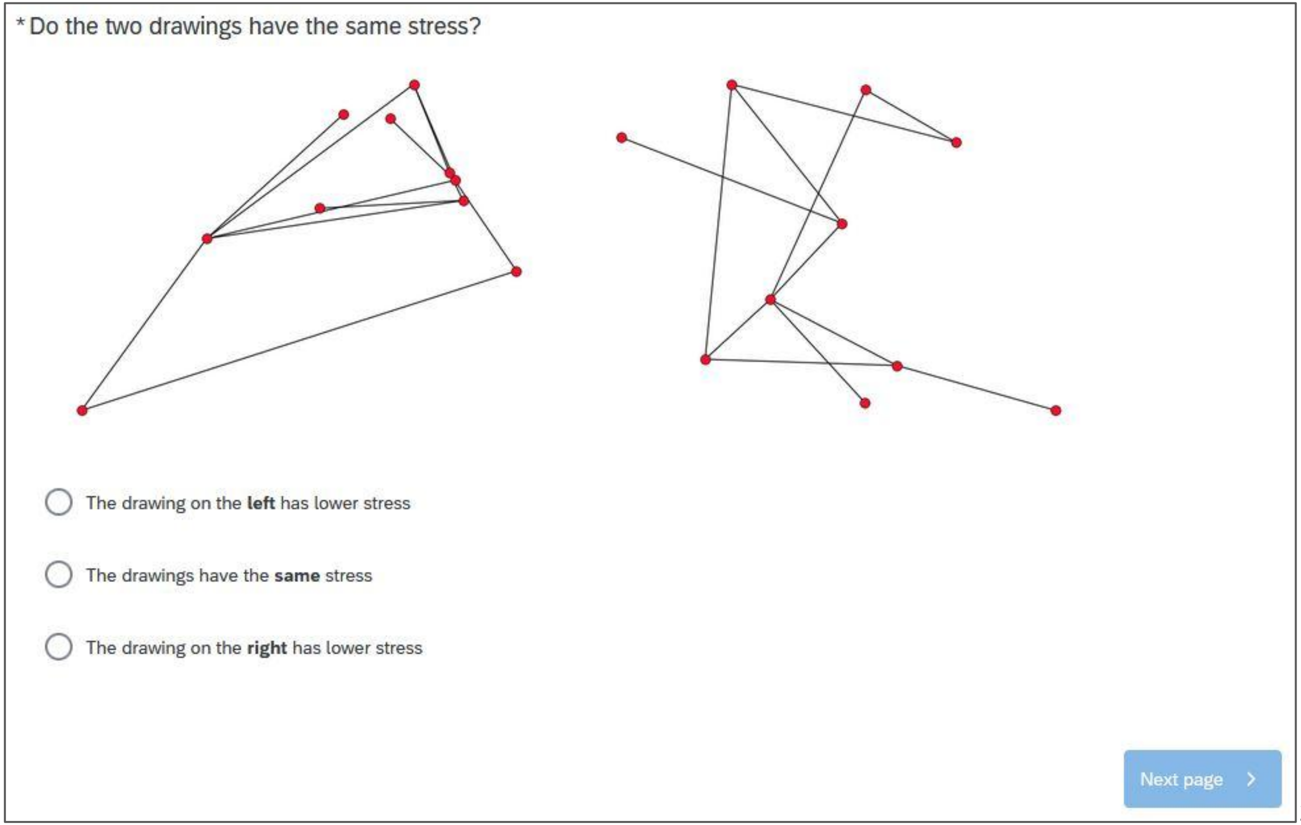
	0.4	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**
0.4	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.45	0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35
0.5	0.1	0.05	0	0.05	0.1	0.15	0.2	0.25	0.3
0.55	0.15	0.1	0.05	0	0.05	0.1	0.15	0.2	0.25
0.6	0.2	0.15	0.1	0.05	0	0.05	0.1	0.15	0.2
0.65	0.25	0.2	0.15	0.1	0.05	0	0.05	0.1	0.15
0.7	0.3	0.25	0.2	0.15	0.1	0.05	0	0.05	0.1
0.75	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0	0.05
0.8	0.4	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0

	0.4	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**
0.4	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.45	0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35
0.5	0.1	0.05	0	0.05	0.1	0.15	0.2	0.25	0.3
0.55	0.15	0.1	0.05	0	0.05	0.1	0.15	0.2	0.25
0.6	0.2	0.15	0.1	0.05	0	0.05	0.1	0.15	0.2
0.65	0.25	0.2	0.15	0.1	0.05	0	0.05	0.1	0.15
0.7	0.3	0.25	0.2	0.15	0.1	0.05	0	0.05	0.1
0.75	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0	0.05
0.8	0.4	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0

	0.4	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**
0.4	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
0.45	0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35
0.5	0.1	0.05	0	0.05	0.1	0.15	0.2	0.25	0.3
0.55	0.15	0.1	0.05	0	0.05	0.1	0.15	0.2	0.25
0.6	0.2	0.15	0.1	0.05	0	0.05	0.1	0.15	0.2
0.65	0.25	0.2	0.15	0.1	0.05	0	0.05	0.1	0.15
0.7	0.3	0.25	0.2	0.15	0.1	0.05	0	0.05	0.1
0.75	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0	0.05
0.8	0.4	0.35	0.3	0.25	0.2	0.15	0.1	0.05	0

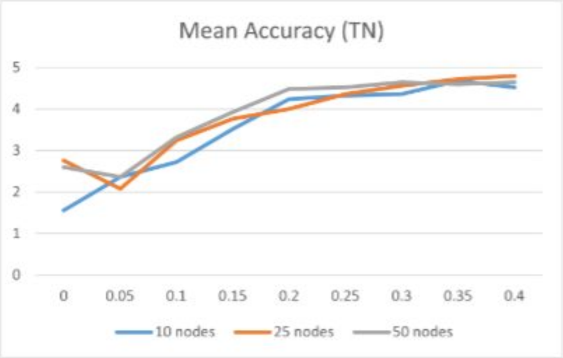
Stimuli - Experimental procedure per participant

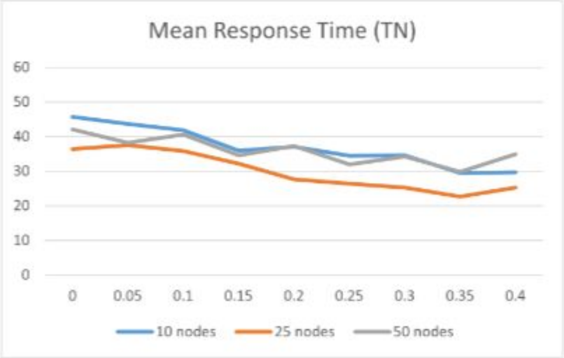
- 45 trials
  - 9 unique KSM differences
  - 5 graphs
- Left/Right chosen randomly
- Exact drawings & KSM values chosen randomly

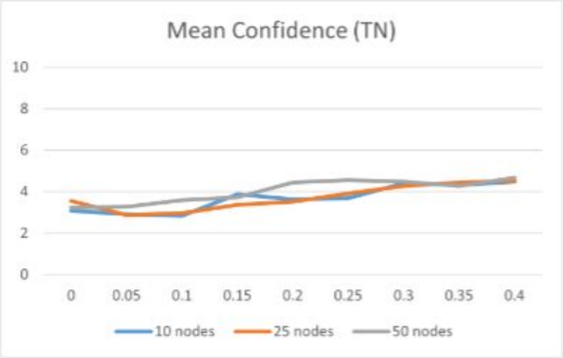


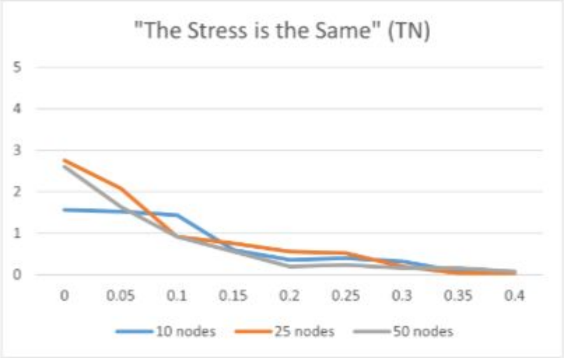
Results - Trained Participants (25 per graph size)

- Participants **performed better** when the stress difference is higher
- Slightly **lower response time** and **greater confidence** for larger differences
- Smaller stress differences were more likely to have the response “The stress is the same”







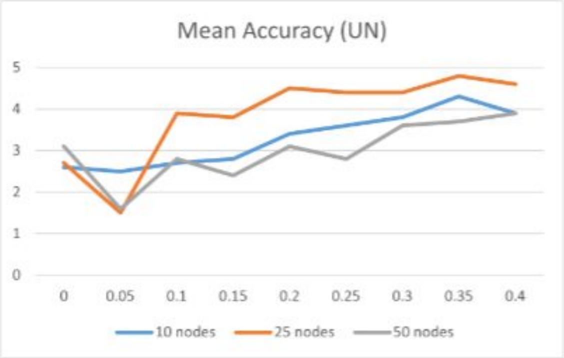


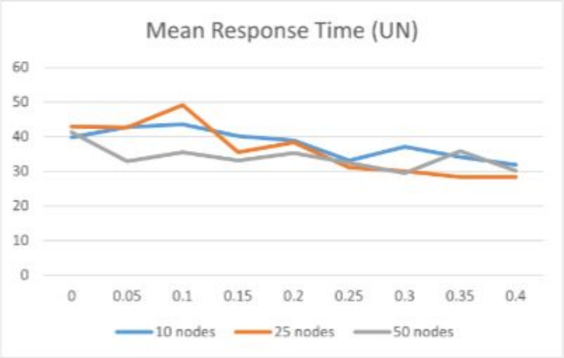
Results - Untrained Participants (10 per graph size)

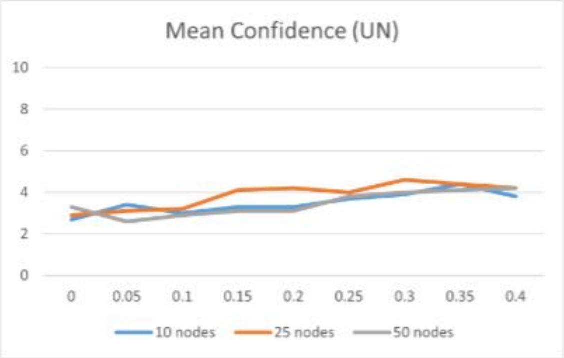
- Participants **performed better** when the stress difference is higher

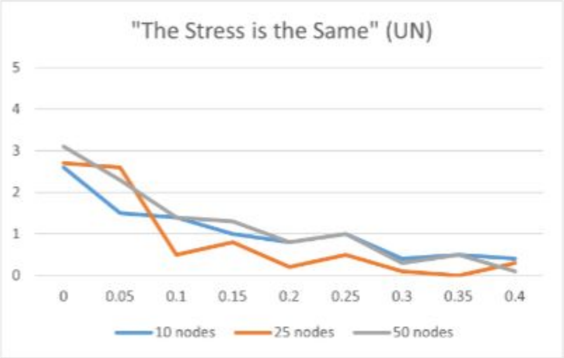
- Slightly **lower response time** and **greater confidence** for larger differences

- Smaller stress differences were more likely to have the response “The stress is the same”







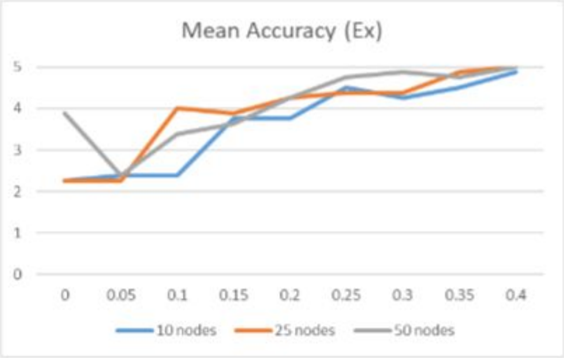


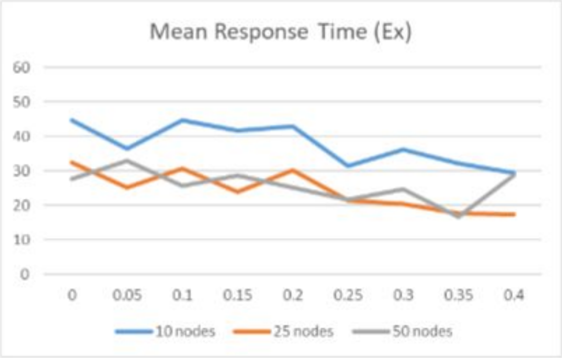
Results - Expert Participants (8 for all graph sizes)

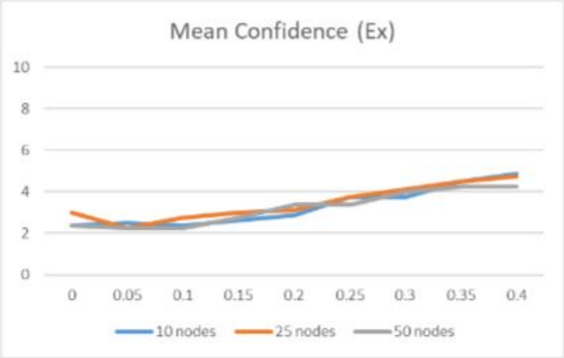
- Participants **performed better** when the stress difference is higher

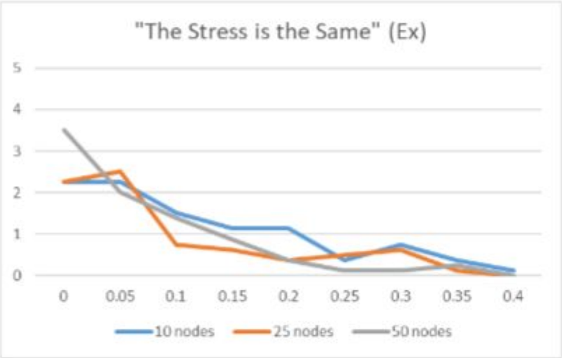
- Slightly **lower response time** and **greater confidence** for larger differences

- Smaller stress differences were more likely to have the response “The stress is the same”





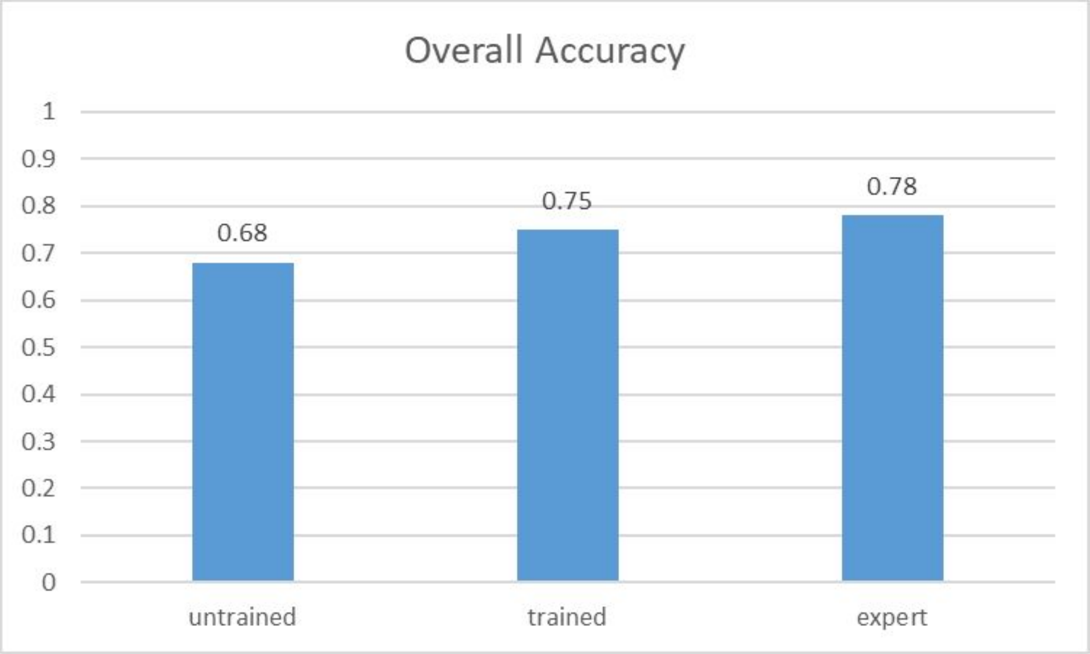




Results - Participant comparisons

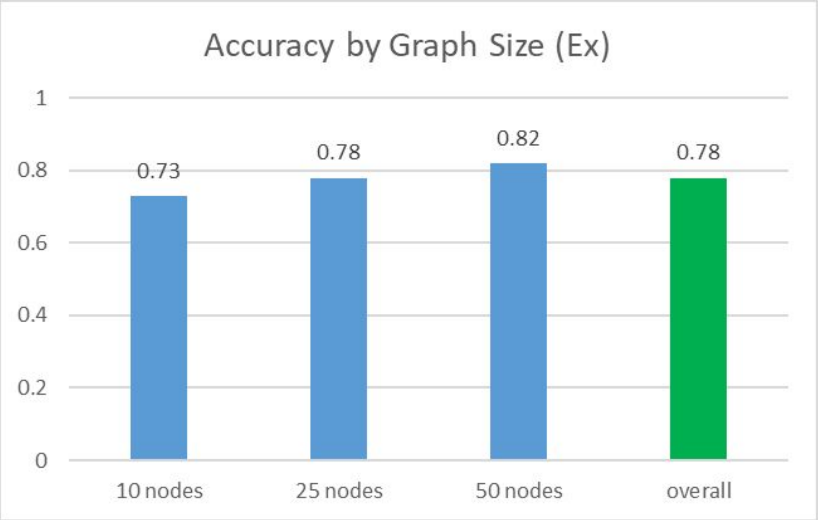
- Independent samples t-test
  - Mean accuracy
    - Trained vs Untrained
    - 0.009
    - Significant difference

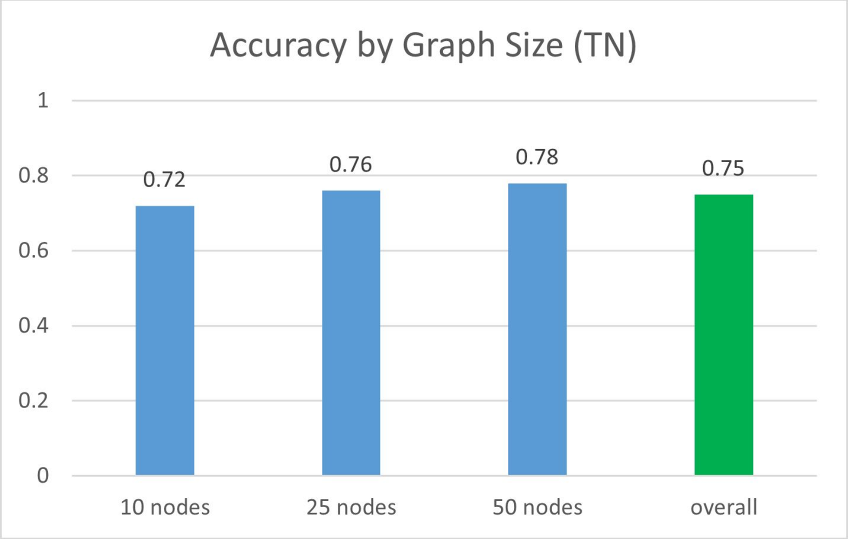
    - Trained vs Expert
    - 0.361
    - No significant difference

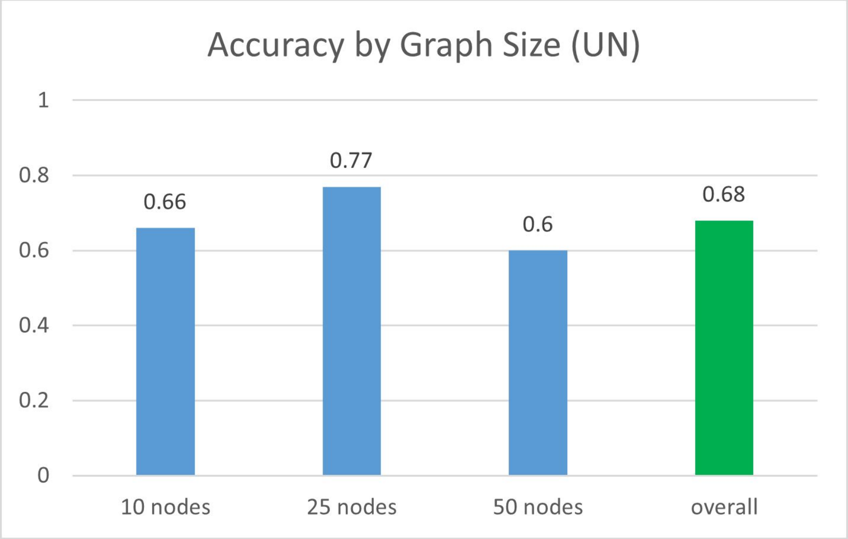


Results - Graph size comparisons

- Graph size made no difference on performance



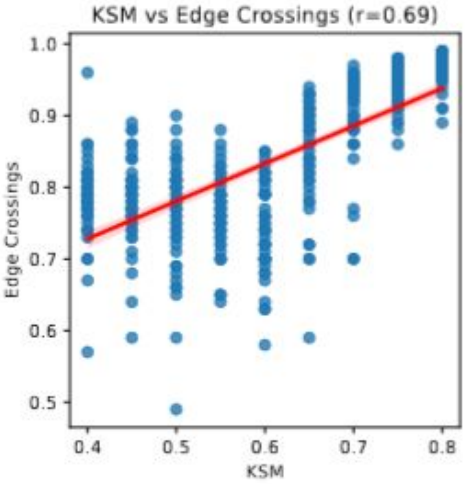




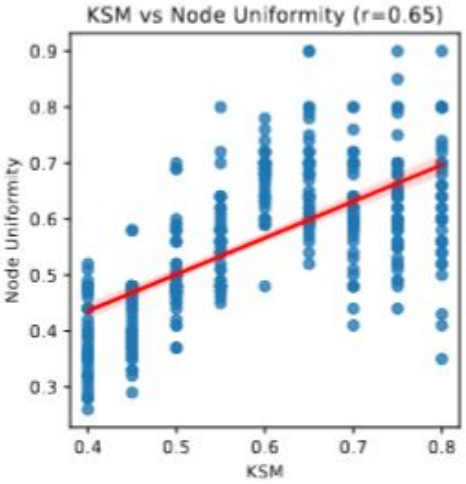
Visual Proxies

“Please describe the strategy you used to determine which drawing had lower stress.”

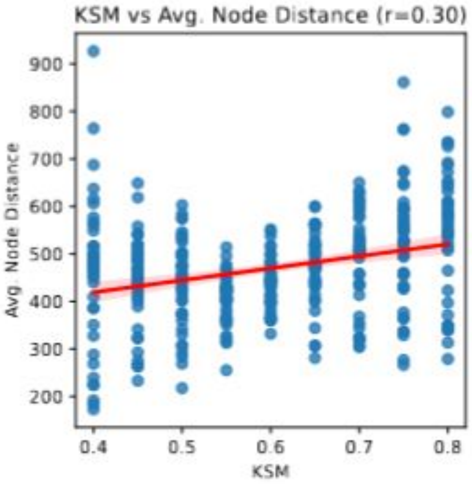
Participants reported several ‘perceptive shortcuts’



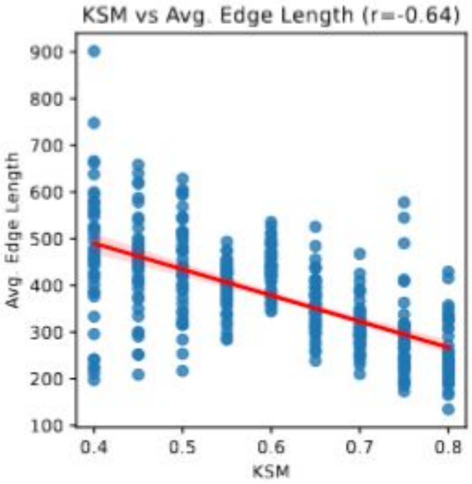
r = 0.69



r = 0.65

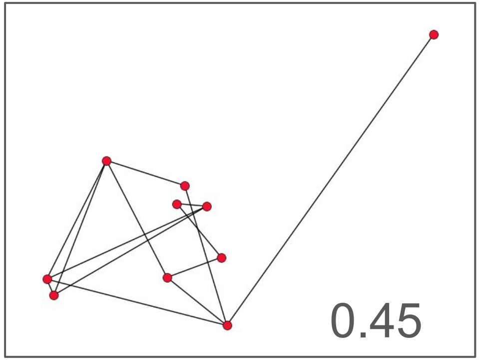


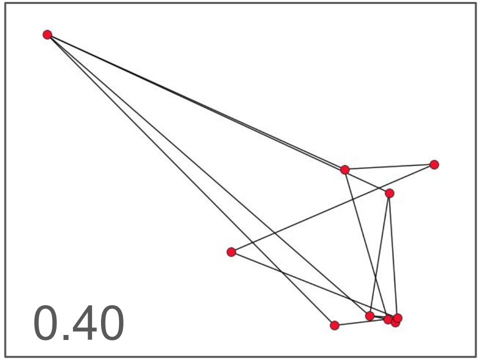
r = 0.30

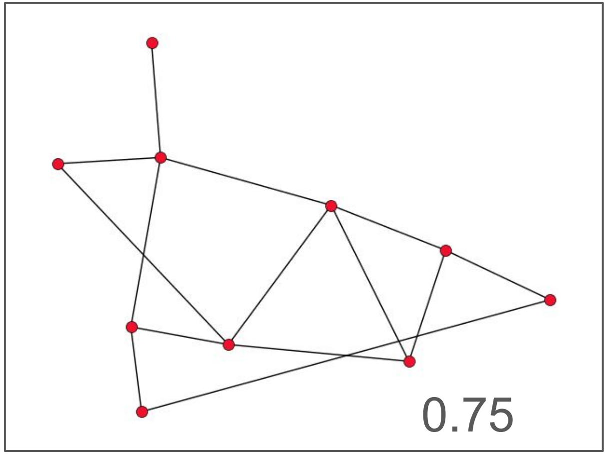


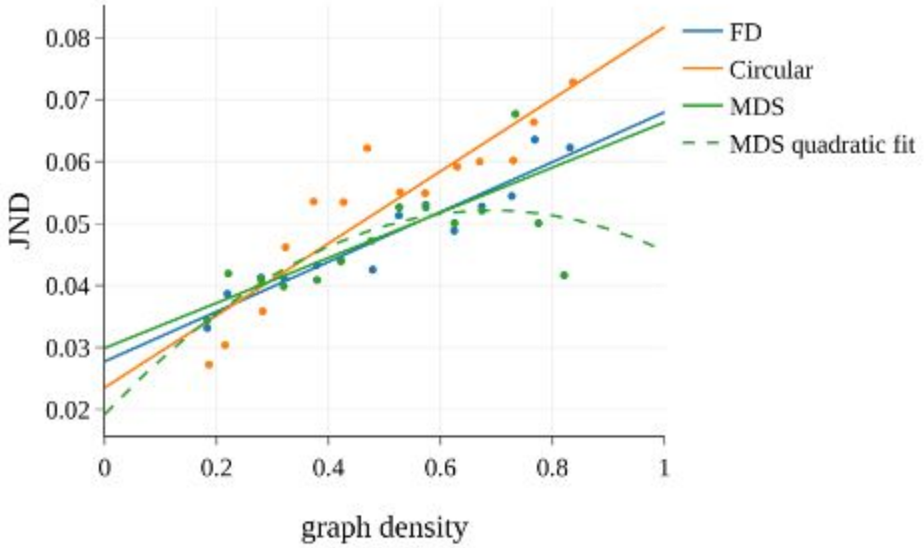
r = - 0.64

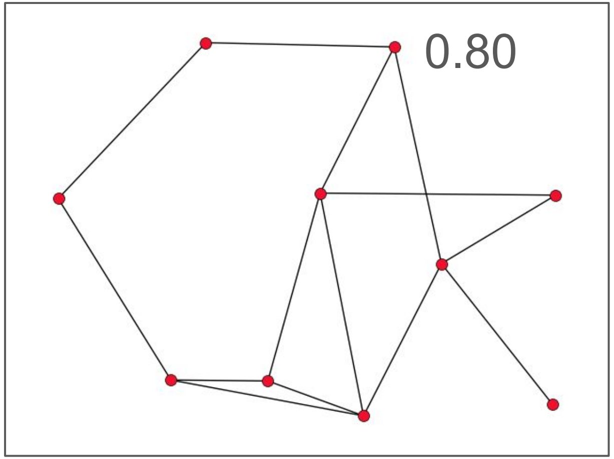
Future work


0.45


0.40


0.75


FD
Circular
MDS
MDS quadratic fit
JND
graph density
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0
0.2
0.4
0.6
0.8
1


0.80

Conclusion

Contact:
Gavin.Mooney@monash.edu

gavjmooney.com

- Contributions:
  - First empirical study testing the perception of stress directly
  - People can be taught to ‘see’ stress as well as experts can
  - Graph size doesn’t seem to affect perception
  - Larger differences in stress are more easily detected
    - Accuracy increases above differences of 0.15